

☺ 3.7 – Substitution and Elimination ☺

Objectives:

1. Solve systems of equations by substitution
2. Explore how the addition and multiplication properties of equations can be used to solve systems of equations by elimination
3. Define inconsistent, consistent, and dependent systems.

Example 1: Solve this system for x and y .

$$y = 15 + 8x$$

$$-10x - 5y = -30$$

$$-10x - 5(15 + 8x) = -30$$

$$-10x - 75 - 40x = -30$$

$$-50x - 75 = -30$$

$$\begin{array}{r} -50x = 45 \\ \underline{-50} \quad \underline{-50} \end{array}$$

$$x = -\frac{9}{10}$$

$$y = 15 + 8\left(-\frac{9}{10}\right)$$

$$y = 15 - 7.2$$

$$y = 7.8$$

$(-\frac{9}{10}, 7.8)$

Example 2: Solve this system.

$$3x - 4y = 8$$

$$y = x - 1$$

$$3x - 4(x - 1) = 8$$

$$3x - 4x + 4 = 8$$

$$-1x + 4 = 8$$

$$-1x = 4$$

$$x = -4$$

$$y = -4 - 1$$

$$y = -5$$

$(-4, -5)$

Example 3: Solve the following systems:

$$4x + 3y = 14$$

$$3x - 3y = 14$$

$$\begin{array}{r} 7x = 28 \\ \underline{7} \quad \underline{7} \end{array}$$

$$x = 4$$

$$4(4) + 3y = 14$$

$$16 + 3y = 14$$

$$3y = -2$$

$$y = -\frac{2}{3}$$

$(4, -\frac{2}{3})$

$$-2x + 5y = 6$$

$$2x + y = 6$$

$$6y = 12$$

$$y = 2$$

$$2x + 2 = 6$$

$$2x = 4$$

$$x = 2$$

$(2, 2)$

Example 4: Solve the system:

$$8x - 8y = 8$$

$$(-10x + 4y = -7) \cdot 2$$

$$\begin{array}{r} 8x - 8y = 8 \\ -20x + 8y = -14 \\ \hline -12x = -6 \end{array}$$

$$x = \frac{1}{2}$$

$$8\left(\frac{1}{2}\right) - 8y = 8$$

$$4 - 8y = 8$$

$$-8y = 4$$

$$y = -\frac{1}{2}$$

$$\left(\frac{1}{2}, -\frac{1}{2}\right)$$

Investigation • What's Your System?

In this investigation you will discover different classifications of systems and their properties.

Step 1: Use the method of *elimination* to solve each system. (Don't be surprised if it doesn't seem to work). Then graph the system of equations.

a. $(2x + 8y = 16) \cdot -1$ $(8, 0) (0, 2)$
 $2x - 2y = 14$ $(7, 0) (0, -7)$

$$-2x - 8y = -16$$

$$-10y = -2$$

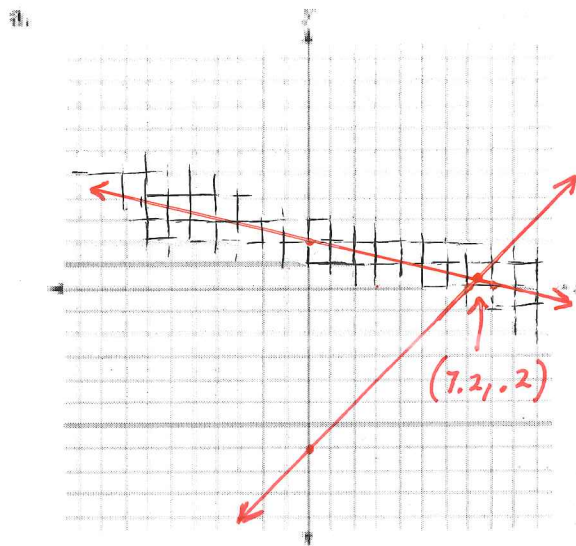
$$y = \frac{1}{5}$$

$$2x - 2\left(\frac{1}{5}\right) = 14$$

$$2x - \frac{2}{5} = 14$$

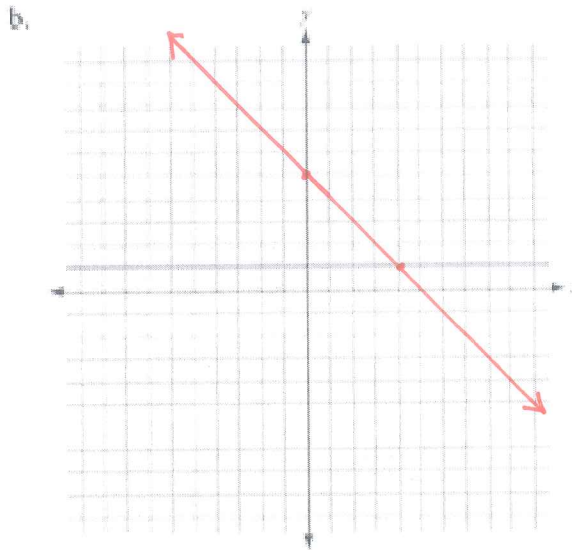
$$2x = 14.4$$

$$x = 7.2$$



$$\begin{array}{l}
 \text{b. } \begin{cases} 3x + 3y = 12 \\ -6x - 6y = -24 \end{cases} \quad \begin{matrix} (0,4) (4,0) \\ (0,4) (4,0) \end{matrix} \\
 \hline
 6x + 6y = 24 \\
 0 = 0
 \end{array}$$

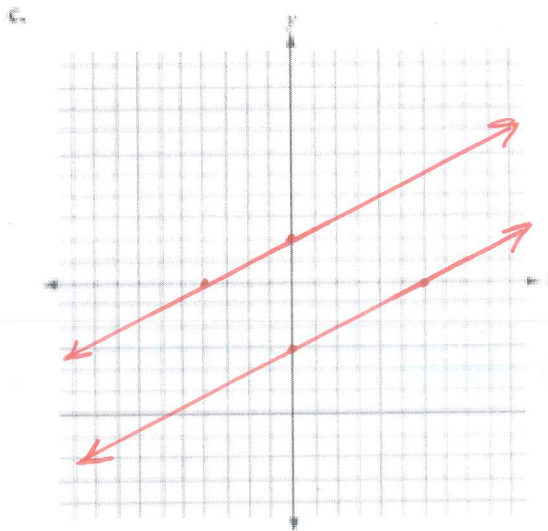
TRUE
INFINITELY
MANY
SOLUTIONS



SAME LINE

$$\begin{array}{l}
 \text{c. } \begin{cases} 3(4x - 8y = 24) \\ 4(-3x + 6y = 12) \end{cases} \quad \begin{matrix} (0,-3) (6,0) \\ (0,2) (-4,0) \end{matrix} \\
 \hline
 12x - 24y = 72 \\
 -12x + 24y = 48 \\
 \hline
 0 \neq 120
 \end{array}$$

FALSE
NO SOLUTION



PARALLEL

A system that has a solution (a point or points of intersection) is called **consistent**.

Which of the three previous systems are consistent?

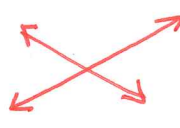

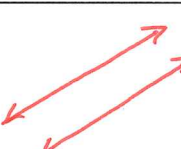
A system that has no solution is called **inconsistent**.

Which of the previous systems are inconsistent?

A system that has infinitely many solutions is called **dependent**.

For linear systems this means the equations are equivalent (though they may look identical).

A system that has a one solution is called **independent**.

Classification	An example of what the graph looks like	Result when using the elimination method
Consistent Independent		$x = \#$ $y = \#$
Consistent Dependent		VARIABLES DROP OUT TRUE STATEMENT $12 = 12$ $0 = 0$
Inconsistent		VARIABLES DROP OUT FALSE STATEMENT $0 \neq 13$ $10 \neq 48$